12.6 Surface Area and Volume of Spheres

What *you should learn*

GOAL 1 Find the surface area of a sphere.

GOAL 2 Find the volume of a sphere in **real life**, such as the ball bearing in **Example 4**.

Why *you should learn it*

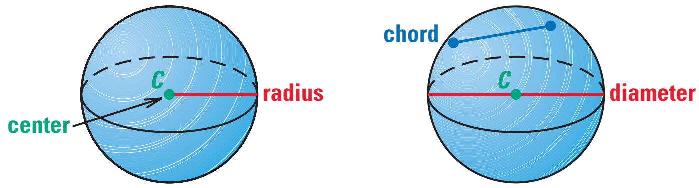
▼ You can find the surface area and volume of **real-life** spherical objects, such as the planets and moons in **Exs. 18 and 19**.

REAL LIFE

Saturn

GOAL 1 FINDING THE SURFACE AREA OF A SPHERE

In Lesson 10.7, a circle was described as the locus of points in a plane that are a given distance from a point. A **sphere** is the locus of points in *space* that are a given distance from a point. The point is called the **center of the sphere**. A **radius of a sphere** is a segment from the center to a point on the sphere.

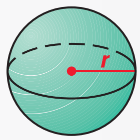


A **chord of a sphere** is a segment whose endpoints are on the sphere. A **diameter** is a chord that contains the center. As with circles, the terms radius and diameter also represent distances, and the diameter is twice the radius.

THEOREM

THEOREM 12.11 ***Surface Area of a Sphere***

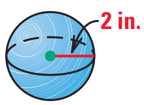
The surface area S of a sphere with radius r is $S = 4\pi r^2$.



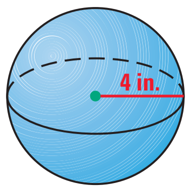
EXAMPLE 1 *Finding the Surface Area of a Sphere*

Find the surface area. When the radius doubles, does the surface area double?

a.



b.



SOLUTION

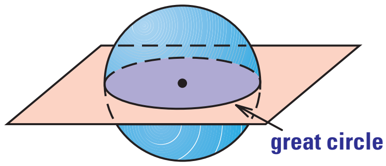
a. $S = 4\pi r^2 = 4\pi(2)^2 = 16\pi \text{ in.}^2$

b. $S = 4\pi r^2 = 4\pi(4)^2 = 64\pi \text{ in.}^2$

The surface area of the sphere in part (b) is four times greater than the surface area of the sphere in part (a) because $16\pi \cdot 4 = 64\pi$.

▶ So, when the radius of a sphere doubles, the surface area does *not* double.

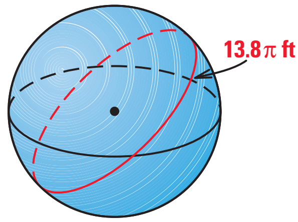
If a plane intersects a sphere, the intersection is either a single point or a circle. If the plane contains the center of the sphere, then the intersection is a **great circle** of the sphere. Every great circle of a sphere separates a sphere into two congruent halves called **hemispheres**.



EXAMPLE 2 *Using a Great Circle*

STUDENT HELP
INTERNET
HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for extra examples.

The circumference of a great circle of a sphere is 13.8π feet. What is the surface area of the sphere?



SOLUTION

Begin by finding the radius of the sphere.

$C = 2\pi r$	Formula for circumference of circle
$13.8\pi = 2\pi r$	Substitute 13.8π for C.
$6.9 = r$	Divide each side by 2π.

Using a radius of 6.9 feet, the surface area is

$$S = 4\pi r^2 = 4\pi(6.9)^2 = 190.44\pi \text{ ft}^2.$$

▶ So, the surface area of the sphere is 190.44π, or about 598 ft^2.

EXAMPLE 3 *Finding the Surface Area of a Sphere*

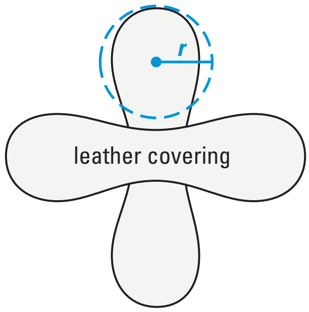
REAL LIFE

BASEBALL A baseball and its leather covering are shown. The baseball has a radius of about 1.45 inches.

a. Estimate the amount of leather used to cover the baseball.

b. The surface of a baseball is sewn from two congruent shapes, each of which resembles two joined circles. How does this relate to the formula for the surface area of a sphere?

$r = 1.45$ in.



SOLUTION

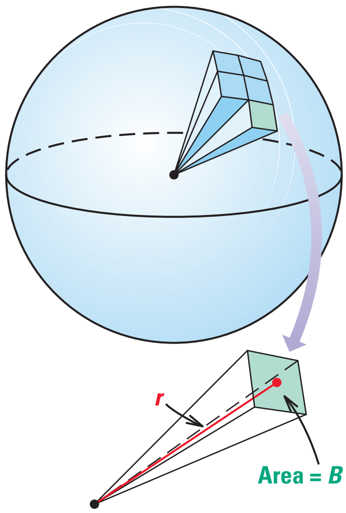
a. Because the radius r is about 1.45 inches, the surface area is as follows:

$S = 4\pi r^2$	Formula for surface area of sphere
$\approx 4\pi(1.45)^2$	Substitute 1.45 for r.
$\approx 26.4 \text{ in.}^2$	Use a calculator.

b. Because the covering has two pieces, each resembling two joined circles, then the entire covering consists of four circles with radius r. The area of a circle of radius r is $A = \pi r^2$. So, the area of the covering can be approximated by $4\pi r^2$. This is the same as the formula for the surface area of a sphere.

GOAL 2 FINDING THE VOLUME OF A SPHERE

Imagine that the interior of a sphere with radius r is approximated by n pyramids, each with a base area of B and a height of r, as shown. The volume of each pyramid is $\frac{1}{3}Br$ and the sum of the base areas is nB. The surface area of the sphere is approximately equal to nB, or $4\pi r^2$. So, you can approximate the volume V of the sphere as follows.



$V \approx n\frac{1}{3}Br$ — Each pyramid has a volume of $\frac{1}{3}Br$.

$= \frac{1}{3}(nB)r$ — Regroup factors.

$\approx \frac{1}{3}(4\pi r^2)r$ — Substitute $4\pi r^2$ for nB.

$= \frac{4}{3}\pi r^3$ — Simplify.

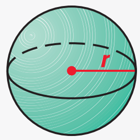
STUDENT HELP

Study Tip
If you understand how a formula is derived, then it will be easier for you to remember the formula.

THEOREM

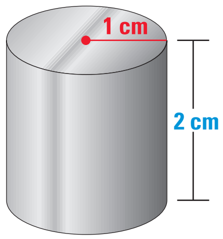
THEOREM 12.12 ***Volume of a Sphere***

The volume V of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.

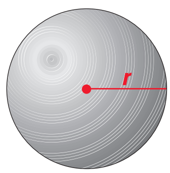


EXAMPLE 4 *Finding the Volume of a Sphere*

BALL BEARINGS To make a steel ball bearing, a cylindrical *slug* is heated and pressed into a spherical shape with the same volume. Find the radius of the ball bearing below.



slug



ball bearing

SOLUTION

To find the volume of the slug, use the formula for the volume of a cylinder.

$$V = \pi r^2 h = \pi(1^2)(2) = 2\pi \text{ cm}^3$$

To find the radius of the ball bearing, use the formula for the volume of a sphere and solve for r.

$V = \frac{4}{3}\pi r^3$ — Formula for volume of sphere

$2\pi = \frac{4}{3}\pi r^3$ — Substitute 2π for V.

$6\pi = 4\pi r^3$ — Multiply each side by 3.

$1.5 = r^3$ — Divide each side by 4π.

$1.14 \approx r$ — Use a calculator to take the cube root.

▶ So, the radius of the ball bearing is about 1.14 centimeters.

FOCUS ON PEOPLE

REAL LIFE **IN-LINE SKATING** Ball bearings help the wheels of an in-line skate turn smoothly. The two brothers above, Scott and Brennan Olson, pioneered the design of today's in-line skates.

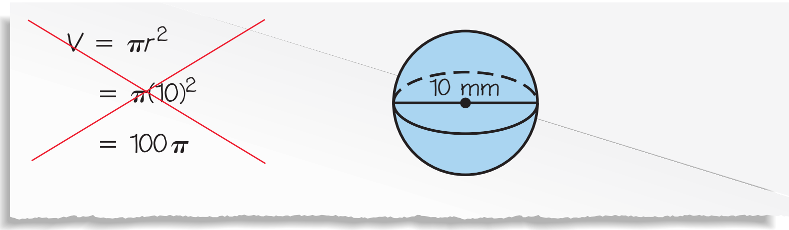
GUIDED PRACTICE

Vocabulary Check ✓

1. The locus of points in space that are __?__ from a __?__ is called a sphere.

Concept Check ✓

2. **ERROR ANALYSIS** Melanie is asked to find the volume of a sphere with a diameter of 10 millimeters. Explain her error(s).



Skill Check ✓

In Exercises 3–8, use the diagram of the sphere, whose center is *P*.

3. Name a chord of the sphere.

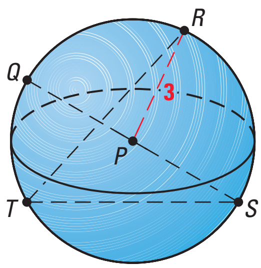
4. Name a segment that is a radius of the sphere.

5. Name a segment that is a diameter of the sphere.

6. Find the circumference of the great circle that contains *Q* and *S*.

7. Find the surface area of the sphere.

8. Find the volume of the sphere.



9. **CHEMISTRY** A helium atom is approximately spherical with a radius of about 0.5×10^{-8} centimeter. What is the volume of a helium atom?

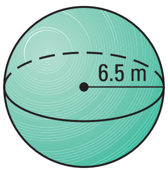
PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 826.

FINDING SURFACE AREA **Find the surface area of the sphere. Round your result to two decimal places.**

10.



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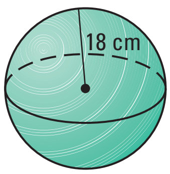

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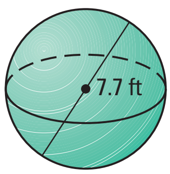


STUDENT HELP

HOMEWORK HELP
Example 1: Exs. 10–12
Example 2: Exs. 13–16
Example 3: Ex. 17
Example 4: Exs. 20–22, 41–43

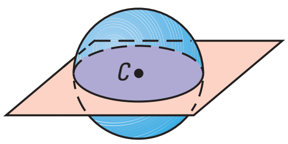
USING A GREAT CIRCLE **In Exercises 13–16, use the sphere below. The center of the sphere is *C* and its circumference is 7.4π inches.**

13. What is half of the sphere called?

14. Find the radius of the sphere.

15. Find the diameter of the sphere.

16. Find the surface area of half of the sphere.



17. **SPORTS** The diameter of a softball is 3.8 inches. Estimate the amount of leather used to cover the softball.

FOCUS ON APPLICATIONS

PLANETS Jupiter is the largest planet in our solar system. It has a diameter of 88,730 miles, or 142,800 kilometers.

APPLICATION LINK
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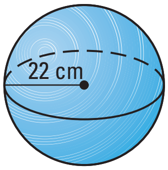
18. **PLANETS** The circumference of Earth at the equator (great circle of Earth) is 24,903 miles. The diameter of the moon is 2155 miles. Find the surface area of Earth and of the moon to the nearest hundred. How does the surface area of the moon compare to the surface area of Earth?

19. **DATA COLLECTION** Research to find the diameters of Neptune and its two moons, Triton and Nereid. Use the diameters to find the surface area of each.

FINDING VOLUME **Find the volume of the sphere. Round your result to two decimal places.**

20.


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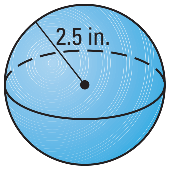

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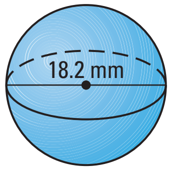
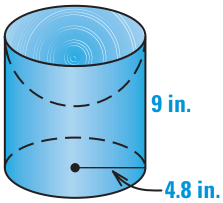
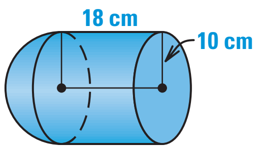


USING A TABLE **Copy and complete the table below. Leave your answers in terms of π.**

	Radius of sphere	Circumference of great circle	Surface area of sphere	Volume of sphere
23.	7 mm	?	?	?
24.	?	?	144π in.^{2}	?
25.	?	10π cm	?	?
26.	?	?	?	$\frac{4000\pi}{3}$ m^3

COMPOSITE SOLIDS **Find (a) the surface area of the solid and (b) the volume of the solid. The cylinders and cones are right. Round your results to two decimal places.**

27.


28.


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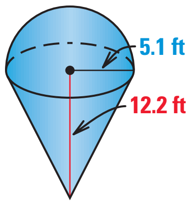


TECHNOLOGY **In Exercises 30–33, consider five spheres whose radii are 1 meter, 2 meters, 3 meters, 4 meters, and 5 meters.**

30. Find the volume and surface area of each sphere. Leave your results in terms of π.

31. Use your answers to Exercise 30 to find the ratio of the volume to the surface area, $\frac{V}{S}$, for each sphere.

32. Use a graphing calculator to plot the graph of $\frac{V}{S}$ as a function of the radius. What do you notice?

33. *Writing* If the radius of a sphere triples, does its surface area triple? Explain your reasoning.

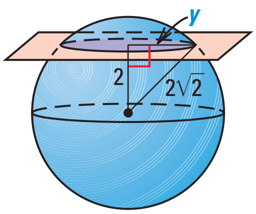
34. **VISUAL THINKING** A sphere with radius r is inscribed in a cylinder with height $2r$. Make a sketch and find the volume of the cylinder in terms of r.

STUDENT HELP

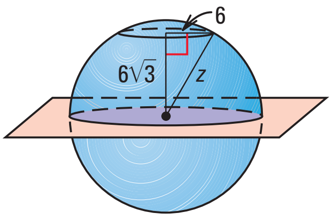
HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for help with Exs. 35 and 36.

USING ALGEBRA **In Exercises 35 and 36, solve for the variable. Then find the area of the intersection of the sphere and the plane.**

35.



36.



37. **CRITICAL THINKING** Sketch the intersection of a sphere and a plane that does not pass through the center of the sphere. If you know the circumference of the circle formed by the intersection, can you find the surface area of the sphere? Explain.

SPHERES IN ARCHITECTURE **The spherical building below has a diameter of 165 feet.**

38. Find the surface area of a sphere with a diameter of 165 feet. Looking at the surface of the building, do you think its surface area is the same? Explain.

39. The surface of the building consists of 1000 (nonregular) triangular pyramids. If the lateral area of each pyramid is about 267.3 square feet, estimate the actual surface area of the building.

40. Estimate the volume of the building using the formula for the volume of a sphere.

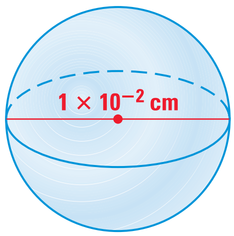
BALL BEARINGS **In Exercises 41–43, refer to the description of how ball bearings are made in Example 4 on page 761.**

41. Find the radius of a steel ball bearing made from a cylindrical slug with a radius of 3 centimeters and a height of 6 centimeters.

42. Find the radius of a steel ball bearing made from a cylindrical slug with a radius of 2.57 centimeters and a height of 4.8 centimeters.

43. If a steel ball bearing has a radius of 5 centimeters, and the radius of the cylindrical slug it was made from was 4 centimeters, then what was the height of the cylindrical slug?

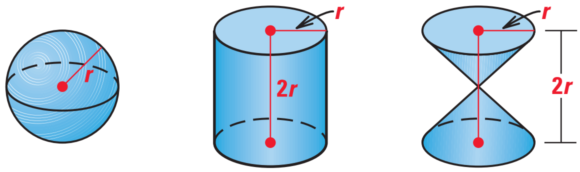
44. **COMPOSITION OF ICE CREAM** In making ice cream, a mix of solids, sugar, and water is frozen. Air bubbles are whipped into the mix as it freezes. The air bubbles are about 1×10^{-2} centimeter in diameter. If one quart, 946.34 cubic centimeters, of ice cream has about 1.446×10^9 air bubbles, what percent of the ice cream is air? (*Hint:* Start by finding the volume of one air bubble.)



Air bubble

Test Preparation

MULTI-STEP PROBLEM **Use the solids below.**



45. Write an expression for the volume of the sphere in terms of r.

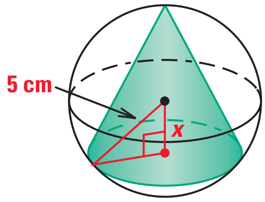
46. Write an expression for the volume of the cylinder in terms of r.

47. Write an expression for the volume of the solid composed of two cones in terms of r.

48. Compare the volumes of the cylinder and the cones to the volume of the sphere. What do you notice?

★ Challenge

49. A cone is inscribed in a sphere with a radius of 5 centimeters, as shown. The distance from the center of the sphere to the center of the base of the cone is x. Write an expression for the volume of the cone in terms of x. (*Hint:* Use the radius of the sphere as part of the height of the cone.)



EXTRA CHALLENGE
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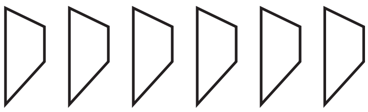
MIXED REVIEW

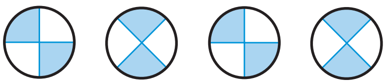
CLASSIFYING PATTERNS **Name the isometries that map the frieze pattern onto itself.** **(Review 7.6)**

50.



51.

52.



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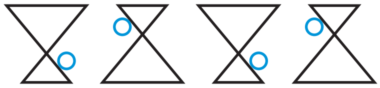
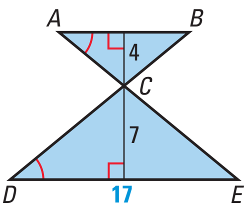


FINDING AREA **In Exercises 54–56, determine whether $\triangle ABC$ is similar to $\triangle EDC$. If so, then find the area of $\triangle ABC$.** **(Review 8.4, 11.3 for 12.7)**

54.



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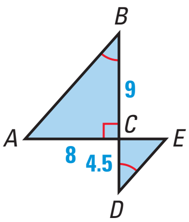

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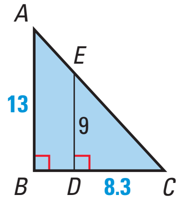

57. MEASURING CIRCLES The tire at the right has an outside diameter of 26.5 inches. How many revolutions does the tire make when traveling 100 feet? **(Review 11.4)**