Unit 2: Reasoning with Equations and Inequalities

Unit 2 focuses on equations and inequalities. From transforming equations or inequalities to graphing their solutions, this unit covers linear relationships with one or two variables. Familiarity with the properties of operations and equality is essential for mastering the skills and concepts covered in this unit. This unit builds on your knowledge of coordinates and extends it to the use of algebraic methods to solve systems of equations.

KEY STANDARDS

Understand solving equations as a process of reasoning and explain the reasoning

MCC9-12.A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. (*Students should focus on and master linear equations and be able to extend and apply their reasoning to other types of equations in future courses*.)

Solve equations and inequalities in one variable

MCC9-12.A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. (*Extend earlier work with solving linear equations to solving linear inequalities in one variable and to solving literal equations that are linear in the variable being solved for. Include simple exponential equations that rely only on application of the laws of exponents, such as* $5^x = 125$ *or* $2^x = 1/16$.)

Solve systems of equations

MCC9-12.A.REI.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. (*Limit to linear systems*.)

MCC9-12.A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Represent and solve equations and inequalities graphically

MCC9-12.A.REI.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

TRANSFORMATIONS OF EQUATIONS AND INEQUALITIES

KEY IDEAS

1. ***Equivalent expressions*** produce the same result when substituting values for variables.

   Example:
   Is the expression $\frac{6x+8}{2}$ equivalent to $3x+4$?

   Solution:
   Yes, the expressions are equivalent.

$\frac{6x+8}{2}$	Write the original expression.
$\frac{2(3x+4)}{2}$	Factor or use the distributive property (reversed).
$3x+4$	Simplify.

2. Manipulating an equation or inequality to show an equivalent relationship is often necessary to solve a problem. Often the variable in an equation or inequality is isolated. Here is a list of what you may do to correctly transform an equation or inequality in order to preserve its solution.

   - Substitution: Replace a quantity, including entire expressions, with an equivalent quantity. (See Key Idea 1.)
   - Use the properties of equality or inequality.

Name of Property	Procedure	Valid for	
		Equations	Inequalities
Addition	Add the same positive or negative number to both sides.	✓	✓
Subtraction	Subtract the same positive or negative number to both sides.	✓	✓
Multiplication	Multiply both sides by the same positive or negative number (not 0).	✓	✓ for positive numbers only. Multiplication by a negative number reverses the inequality sign.
Division	Divide both sides by the same positive or negative number (not 0).	✓	✓ for positive numbers only. Division by a negative number reverses the inequality sign.
Raising to a power	Raise both sides to the same power.	✓, but may introduce an erroneous solution.	✓, but may introduce erroneous solutions.
Taking a root	Take the same root of both sides	✓, but may cause the loss of a solution.	✓, but may cause the loss of a solutions.

- Use the Reflexive and Transitive properties of Equality and Inequality.
- Use the Symmetric Property of Equality.

Below are examples of when you might need to transform an equation.

Example:

Solve the equation $2y+4=3(2x-6)$ for y. Show and justify your steps.

Solution:

$2y+4=3(2x-6)$	Write the original equation.
$2y+4=6x-18$	Use the Distributive Property to write an equivalent expression on the right side.
$2y+4-4=6x-18-4$	Subtract 4 from both sides.
$2y=6x-22$	Combine like terms on both sides.
$\frac{2y}{2}=\frac{6x}{2}-\frac{22}{2}$	Divide each term on both sides by 2.
$y=3x-11$	Simplify both sides.

The equation was transformed to isolate the variable y, not to find a numerical answer.

Example:

Solve the equation $14 = ax + 6$ for x. Show and justify your steps.

Solution:

$14 = ax + 6$	Write the original equation.
$14 - 6 = ax + 6 - 6$	Subtract 6 from both sides.
$8 = ax$	Combine like terms on each side.
$\frac{8}{a} = \frac{8ax}{a}$	Divide each side by a.
$x = \frac{8}{a}$	Simplify.

Example:

Solve the inequality $4 - y > 5$ for y. Show and justify your steps.

Solution:

$4 - y > 5$	Write the original inequality.
$4 - 4 - y > 5 - 4$	Subtract 4 from both sides.
$-y > 1$	Combine like terms on both sides.
$(-1)(-y) < (-1)(1)$	Multiply both sides by -1.
$y < -1$	Simplify.

Important Tips

- Know the properties of operations and the order of operations so you can readily simplify algebraic expressions and prove two expressions are equivalent.
- Be familiar with the properties of equality and inequality. In particular, be aware that when you multiply or divide both sides of an inequality, you must reverse the inequality sign to preserve the relationship.
- Multiply both sides of an equation or inequality by a common denominator as a first step to eliminate denominators.

REVIEW EXAMPLES

1) Are the algebraic expressions $4x^2 - 2x$ and $6x^2 - 2(x^2 - x)$ equivalent?

Solution:

$6x^2 - 2(x^2 - x)$

$6x^2 - 2x^2 + 2x$

$4x^2 + 2x$

No, the expressions are not equivalent because, when simplified, $6x^2 - 2(x^2 - x)$ is $4x^2 + 2x$, not $4x^2 - 2x$.

2) Solve this inequality for y: $6a - 2y > 4$

Solution:

$6a - 2y > 4$	Write the original inequality.
$6a - 6a - 2y > 4 - 6a$	Subtract $6a$ from both sides.
$-2y > 4 - 6a$	Combine like terms on both sides.
$\frac{-2y}{-2} < \frac{4-6a}{-2}$	Divide each side by –2 and reverse the inequality symbol.
$y < -2 + 3a$	Simplify both sides.
$y < 3a - 2$	Write the right side of the expression in standard form.

3) Solve the equation $\frac{m}{6} + \frac{m}{4} = 1$ for m.

Solution:

$\frac{m}{6} + \frac{m}{4} = 1$	Write the original equation.
$2m + 3m = 12$	Multiply both sides by 12.
$5m = 12$	Combine like terms on the left side.
$\frac{5m}{5} = \frac{12}{5}$	Divide each side by 5.
$m = \frac{12}{5}$	Simplify.

EOCT Practice Items

1) Emily wants to solve the equation $ax - w = 3$ for w. Which equation shows the results of a correctly applied strategy?

A. $w = ax - 3$
B. $w = ax + 3$
C. $w = 3 - ax$
D. $w = 3 + ax$

[Key: A]

2) Which equation is equivalent to $\frac{7x}{4} - \frac{3x}{8} = 11$?

A. $17x = 88$
B. $11x = 88$
C. $4x = 44$
D. $2x = 44$

[Key: B]

3) Which equation is equivalent to $4n = 2(t - 3)$ when solved for t?

A. $t = \frac{4n - 2}{3}$

B. $t = \frac{4n - 3}{2}$

C. $t = \frac{4n + 6}{2}$

D. $t = 4n - 3$

[Key: C]

4) Which equation is equivalent to $6(x + 4) = 2(y + 5)$ when solved for y?

A. $y = x + 3$

B. $y = x + 5$

C. $y = 3x + 7$

D. $y = 3x + 17$

[Key: C]

SOLVING EQUATIONS AND INEQUALITIES

KEY IDEAS

1. To solve an equation or inequality means to find the quantity or quantities that make the equation or inequality true. The strategies for solving an equation or inequality depend on the number of variables and the exponents that are included.

2. An algebraic method for solving a linear equation with one variable:

   Write equivalent expressions until the desired variable is isolated on one side. Be careful not to lose any possible answers or introduce incorrect answers.

   Example:

   Solve $2(3-a)=18$.

   Solution:

   Solve the equation two ways:

$$\begin{aligned} 2(3-a)&=18 \\ 3-a&=9 \\ -a&=6 \\ a&=-6 \end{aligned}$$

$$\begin{aligned} 2(3-a)&=18 \\ 6-2a&=18 \\ -2a&=12 \\ a&=-6 \end{aligned}$$

3. An algebraic method for solving a linear inequality with one variable:

Write equivalent expressions until the desired variable is isolated on one side. When the sides of an inequality are interchanged, make sure the inequality sign points at the same expression.

Example:

Solve $2(5-x)>8$ for x.

Solution:

Solve the inequality using either of these two ways:

$$\begin{aligned} 2(5-x)&>8 \\ 5-x&>4 \\ -x&>-1 \\ x&<1 \end{aligned}$$

$$\begin{aligned} 2(5-x)&>8 \\ 10-2x&>8 \\ -2x&>-2 \\ x&<1 \end{aligned}$$

Important Tips

- Be careful when solving inequalities where you have to multiply or divide both sides by a negative number. Remember to change the inequality sign when this occurs.
- Be familiar with the properties of equality and inequality so you can transform equations or inequalities.
- Sometimes eliminating denominators by multiplying all terms by a common denominator makes it easier to solve an equation or inequality.

REVIEW EXAMPLES

1) Karla wants to save up for a prom dress. She figures she can save $9 each week from the money she earns babysitting. If she plans to spend up to $150 for the dress, how many weeks will it take her to save enough money?

Solution:

Let w represent the number of weeks. If she saves \$9 each week, the amount Karla has saved will be $9w$ dollars after w weeks. We need to determine the minimum number of weeks it will take her to save \$150. So, we can use the inequality $9w > 150$ to solve the problem. We need to transform $9w > 150$ to isolate w. So let's divide both sides by 9. We get $w > 16\frac{2}{3}$ weeks. Since we do not know what day Karla gets paid each week, we need the answer to be a whole number. The answer has to be 17, the smallest whole number greater than $16\frac{2}{3}$. We find she will have \$144 after 16 weeks, and \$153 after 17 weeks.

2) Joachim wants to know if he can afford to add texting to his cell phone plan. He currently spends \$21.49 a month for his cell phone plan. The most he can spend for his cell phone is \$30 a month. He could get unlimited texts added to his plan for an additional \$10 each month. Or, he could get a "pay as you go" plan that charges a flat rate of \$0.15 per text message, sent or received. He figures he'll send about two texts per day and receive about three, totaling five text messages per day on average. Should he get text messaging added to his plan or not?

Solution:

Joachim cannot afford either plan. How the solution was determined is below.

At an additional \$10 per month for unlimited texting, Joachim's cell phone bill would be \$31.49 a month. If he chose the pay-as-you-go plan, each day he would expect to pay for 5 text messages. Let t stand for the number of text messages per month. Then, on the pay-as-you-go plan, Joachim could expect his cost to be represented by the expression:

$$\$21.49 + \$0.15t$$
$$21.49 + 0.15t$$

If he must keep his costs at \$30 or less, $21.49 + 0.15t \leq 30$.

We must transform this inequality until we have isolated t.

$21.49 - 21.49 + 0.15t \leq 30 - 21.49$	Subtract 21.49 from both sides.
$0.15t \leq 8.51$	Simplify both sides.
$t \leq 56.733\ldots$	Divide both sides by 0.15.

The transformed inequality tells us that Joachim would need to keep his total texts under 57 per month to afford the pay-as-you-go plan. But, at five texts each day at a minimum of 28 days in a month, the pay-as-you-go plan would cost a minimum of $28 \times 5 \times \$0.15 = \$21$ a month. The total monthly cost for Joachim would be \$42.49. Therefore, neither plan fits Joachim's budget.

3) Two cars, the first traveling 15 miles per hour faster than the second, start at the same point and travel in opposite directions. In 4 hours, they are 300 miles apart. We can use the formula below to determine the rate of the second car.

$$4(r + 15) + 4r = 300$$

What is the rate, r, of the second car?

Solution:

The second car is traveling 30 miles per hour.

$4(r + 15) + 4r = 300$	Write the original equation.
$4r + 60 + 4r = 300$	Multiply 4 by $r + 15$.
$8r + 60 = 300$	Combine like terms.
$8r = 240$	Subtract 60 from each side.
$r = 30$	Divide each side by 8.

EOCT Practice Items

1) This equation can be used to find *h*, the number of hours it takes Flo and Bryan to mow their lawn.

$$\frac{h}{3} + \frac{h}{6} = 1$$

How many hours will it take them?

A. 6
B. 3
C. 2
D. 1

[Key: C]

2) A ferry boat carries passengers back and forth between two communities on the Peachville River.

- **It takes 30 minutes longer for the ferry to make the trip upstream than downstream.**
- **The ferry's average speed in still water is 15 miles per hour.**
- **The river's current is usually 5 miles per hour.**

This equation can be used to determine how many miles apart the two communities are.

$$\frac{m}{15-5}=\frac{m}{15+5}+0.5$$

What is *m*, the distance between communities?

A. 0.5 miles
B. 5 miles
C. 10 miles
D. 15 miles

[Key: C]

3) Which expression represents all values of *x* for which the inequality $\frac{2}{3}+\frac{x}{3}>1$ is true?

A. $x < 1$
B. $x > 1$
C. $x < 5$
D. $x > 5$

[Key: B]

SOLVING A SYSTEM OF TWO LINEAR EQUATIONS

KEY IDEAS

1. A system of linear equations consists of two or more linear equations that may or may not have a common solution. The solution of a system of two linear equations is the set of values for the variables that makes all the equation true. The solutions can be expressed as ordered pairs in coordinate notation (x, y) or as two equations, one for x and the other for y (x = … and y = …).

   Strategies:

   - Use graphs of the equations to visually estimate a common point. First, prepare a table of values for each equation in the system, using the same set of numbers for the first coordinate. Graph both equations and test the coordinates of the point where the lines appear to cross in both equations to see if the coordinates are a common solution.

   Example:

   Solve the system with the equations: $y = 2x - 4$ and $x = y + 1$.

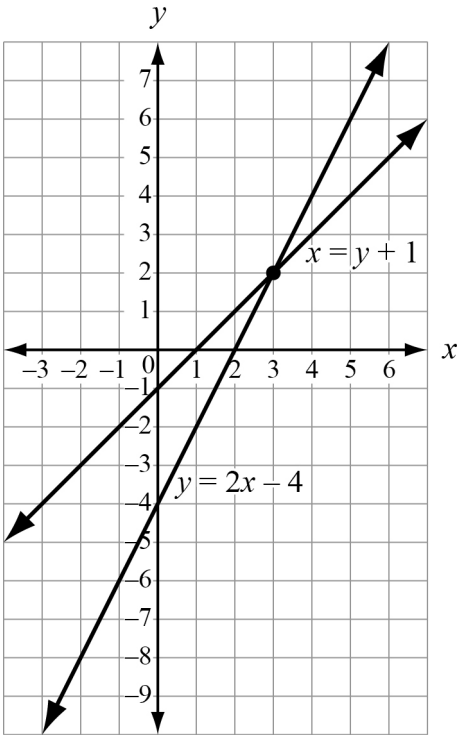
   Solution:

   First, find coordinates of points for each equation. Making a table of values for each is one way to do this. Use the same values for both equations.

x	$2x - 4$
–1	–6
0	–4
1	–2
2	–0
3	2
4	4

x	$y + 1$
–1	–2
0	–1
1	0
2	1
3	2
4	3

Using the tables on the previous page, we display both equations on the graph below.



The graph shows all the ordered pairs of numbers (rows from the table) that satisfy $y = 2x - 4$ and the ordered pairs that satisfy $x = y + 1$. From the graph it appears that the lines cross at about (3, 2). We then try that combination in both equations. Indeed, if an x-value of 3 and a y-value of 2 are applied to both equations, they work. That means, once again, we have found at least one combination that works for both equations: $x = 3$, $y = 2$. The graph also suggests that (3, 2) is the only point the lines have in common. That means we have found the only pair of numbers that work for both equations.

- Simplify the problem by eliminating one of the two variables.

Substitution method: Use one equation to isolate a variable and replace that variable in the other equation with the equivalent expression you just found. Solve for the one remaining variable. Use the solution to the remaining variable to find the unknown you eliminated.

Example:

Find the solution to the system with $2x - y = 1$ and $5 - 3x = 2y$.

Solution:

Begin by choosing one of the equations and solving for one of the variables. This variable is the one you will eliminate. We could solve the top equation for y.

$$\begin{aligned} 2x - y &= 1 \\ 2x &= 1 + y \\ 2x - 1 &= y \\ y &= 2x - 1 \end{aligned}$$

Next, use substitution to replace the variable you are eliminating in the other equation.

$$\begin{aligned} 5 - 3x &= 2y \\ 5 - 3x &= 2(2x - 1) \\ 5 - 3x &= 4x - 2 \\ 7x &= 7 \\ x &= 1 \end{aligned}$$

Now, find the corresponding y-value.

$$\begin{aligned} 2x - y &= 1 \\ 2(1) - y &= 1 \\ 2 - y &= 1 \\ -y &= 1 - 2 \\ -y &= -1 \\ y &= 1 \end{aligned}$$

We have determined the solution to be $x = 1$ and $y = 1$, or (1, 1).

<u>Addition method</u>: We eliminate a variable by adding the equations or a transformation of the equations. If necessary, transform the equations so the terms with unknowns align on the left side of the equal sign and the other terms are on the right side. Rewrite the system with equal signs aligned and like terms in columns. Transform the equations of the system so that the coefficient of one of the variables in the upper equation is opposite the coefficient of its counterpart in the lower equation. Add the two equations. Solve for the one remaining variable. Use the solution to the remaining unknown to find the variable you eliminated.

Example:

Find the solution to the system $2x - y = 1$ and $5 - 3x = -y$.

Solution:

First, transform the second equation so that it is in standard form.

$$\begin{aligned} 5 - 3x &= -y \\ -3x + y &= -5 \end{aligned}$$

Decide which variable to eliminate. We can eliminate the y-terms because they are opposites.

$$\begin{aligned} 2x - y &= 1 \\ -3x + y &= -5 \end{aligned}$$

Add the equations, term by term, eliminating y and reducing to one equation. This is an application of the addition property of equality.

$$-x = -4$$

Solve for x first, and then use that value to solve for y.

$$\begin{aligned} (-1)(-x) &= (-1)(-4) \\ x &= 4 \end{aligned}$$

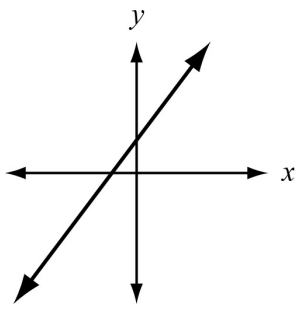
If $2x - y = 1$, then $2(4) - y = 1$. So, $8 - y = 1$, $-y = -7$, and $y = 7$. We get a single solution of $x = 4$ and $y = 7$ as a combination that satisfies both equations.

Important Tips

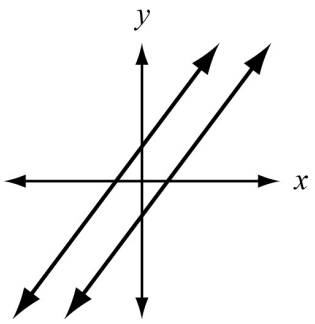
- The graphing method only suggests what a solution of a system might be. Be sure to substitute the ordered pair that appears to be the intersection back into the equations to see if they work.

- When graphing: a) if the lines are parallel, then there is no solution to the system; b) if the lines coincide, then the lines have all their points in common and any pair of points that satisfies one, will satisfy the other.

- When using elimination, if both variables are removed when you try to eliminate one, then the lines coincide. The equations would have all ordered pairs in common. (See figure below.)



- When using elimination, if the result is a false equation such as 3 = 7, that means the system has no solution: the lines would be parallel.



REVIEW EXAMPLES

1) Rebecca has five coins worth 65 cents in her pocket. If she only has quarters and nickels, how many quarters does she have? Use a pair of systems of equations to arrive at your answer and show all steps.

Solution:

If q represents the number of quarters and n represents the number of nickels, the two equations could be $25q + 5n = 65$ (value of quarters plus value of nickels is 65 cents) and $q + n = 5$ (she has 5 coins). The equations in the system would be $25q + 5n = 65$ and $q + n = 5$.

Next, solve $q + n = 5$ for q. By subtracting n from both sides, the result is $q = 5 - n$.

Next, eliminate q by replacing q with $5 - n$ in the other equation: $25(5 - n) + 5n = 65$
Solve this equation for n.

$$25(5-n)+5n=65$$

$$125-25n+5n=65$$

$$125-20n=65$$

$$-20n=-60$$

$$n=3$$

Now solve for q by replacing n with 3 in the equation $q = 5 - n$. So, $q = 5 - 3 = 2$, and 2 is the number of quarters.

Rebecca has 2 quarters and 3 nickels.

2) Peg and Larry purchased "no contract" cell phones. Peg's phone cost \$25 and she pays \$0.25 a minute for calls in the United States. Larry's phone cost \$35 and he pays \$0.20 per minute for calls in the United States. After how many minutes of use will Peg's phone cost more than Larry's?

Solution:

Let x represent the number of minutes of use. Peg's phone costs $25 + 0.25x$. Larry's phone costs $35 + 0.20x$. We want Peg's cost to exceed Larry's: P > L or P – L > 0. If we subtract $35 + 0.20x$ from $25 + 0.25x$, the result is $-10 + 0.05x$. Replacing P – L with $-10 + 0.05x$, we have $-10 + 0.05x > 0$, which we then solve for x.

$$-10 + 0.05x > 0$$

$$-10 + 10 + 0.05x > 0 + 10$$

$$0.05x > 10$$

$$x > 200$$

After 200 phone calls, Peg's phone will cost more than Larry's.

3) Is (3, –1) a solution of this system?

$$y = 2 - x$$
$$3 - 2y = 2x$$

Solution:

Substitute the coordinates of (3, –1) into each equation.

$$y = 2 - x \qquad\qquad 3 - 2y = 2x$$
$$-1 = 2 - 3 \qquad\qquad 3 - 2(-1) = 2(3)$$
$$-1 = -1 \qquad\qquad 3 + 2 = 6$$
$$5 = 6$$

The coordinates of the given point do not satisfy $3 - 2y = 2x$. When you get a false equation trying to solve a system algebraically, it means the equations have no common solution, or that the coordinates of the point are not the solution. So, (3, –1) is not a solution of the system.

4) Solve this system.

$$x - 3y = 6$$
$$-x + 3y = -6$$

Solution:

Add the terms of the equations: 0 + 0 = 0.

The result is always true, so the two equations represent the same line.

5) Solve this system.

$$-3x - y = 10$$
$$3x + y = -8$$

Solution:

Add the terms in the equations: 0 + 0 = 2

The result is never true. The two equations represent parallel lines. As a result, the system has no solution.

EOCT Practice Items

1) A manager is comparing the cost of buying ball caps with the company emblem from two different companies.

- **Company X charges a $50 fee plus $7 per cap.**
- **Company Y charges a $30 fee plus $9 per cap.**

For what number of ball caps will the manager's cost be the same for both companies?

A. 10 caps
B. 20 caps
C. 40 caps
D. 100 caps

[Key: A]

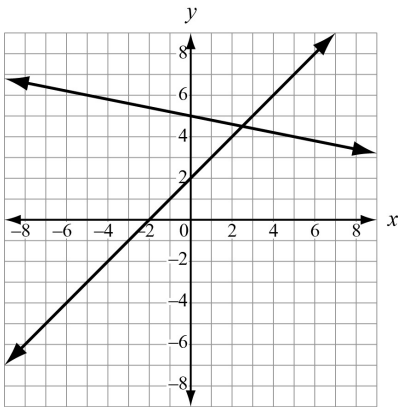
2) A shop sells one-pound bags of peanuts for $2 and three-pound bags of peanuts for $5. If 9 bags are purchased for a total cost of $36, how many three-pound bags were purchased?

A. 3
B. 6
C. 9
D. 18

[Key: B]

3) Which graph would represent a system of linear equations that has multiple common coordinate pairs?

A



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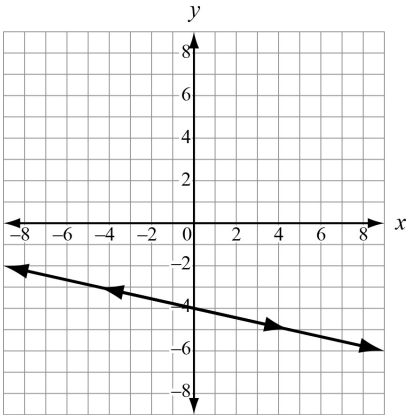

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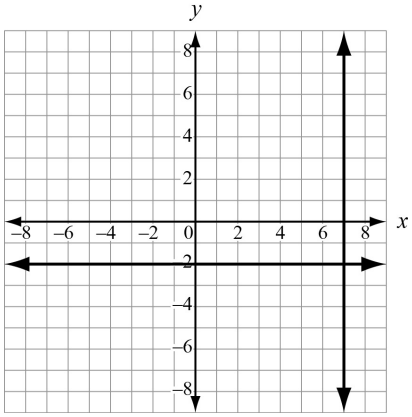

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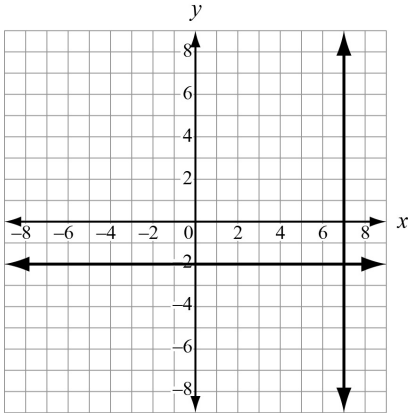


[Key: B]

GRAPHING THE SOLUTIONS OF EQUATIONS AND INEQUALITIES

KEY IDEAS

1. The solution to an equation or inequality can be displayed on a graph using a coordinate or coordinates. If the equation or inequality involves only one variable, then the number line is the coordinate system used.

Example:

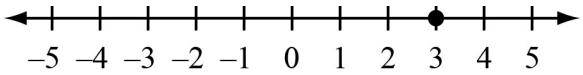
Use a number line to display the solution to $3x + 5 = 14$.

Solution:

First solve the equation:

$$\begin{aligned} 3x+5&=14 \\ 3x+5-5&=14-5 \\ 3x&=9 \\ x&=3 \end{aligned}$$

After transforming the equation we get $x = 3$. Since there was only the variable, x, we will use a number line to display the answer.



For an equation, the display shows the value(s) on the number line that satisfy the equation. Typically a dot is placed on the line where the solution lies.

Example:

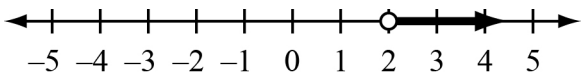
Use a number line to display the solution to $3x + 8 > 14$.

Solution:

First solve the inequality.

$$\begin{aligned} 3x+8&>14 \\ 3x+8-8&>14-8 \\ 3x&>6 \\ x&>2 \end{aligned}$$

After transforming the inequality, the result is $x > 2$. Since there was only the variable x, we will use a number line to display the answer. For an inequality, the display shows the values on the number line that satisfy the inequality. The display is usually a ray drawn on the number line that may or may not include its starting point. If the starting point is not included (<, >), then an open circle is used at that value.



The ray points to the right because the numbers that fit the inequality are greater than 2. The dot at 2 is open because 2 does not fit the inequality, but the numbers that fit start at 2.

Example:

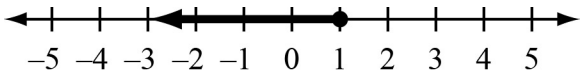
Use a number line to display the solution to $7 - 4x \geq 3$.

Solution:

First solve the inequality:

$$\begin{aligned} 7-4x&\geq 3 \\ 7-4x-7&\geq 3-7 \\ -4x&\geq -4 \\ x&\leq 1 \end{aligned}$$

After transforming the inequality, the result is $x \leq 1$. Since there was only the variable x, we will use a number line to display the answer. If the endpoint is included (≥, ≤), the circle that represents the endpoint is closed.



The ray points to the left because the numbers that fit the inequality are less than or equal to 1. The circle that lies at 1 is closed because 1 does satisfy the inequality.

2. If the equation or inequality involves two variables, then a rectangular coordinate system is used to display the solution. For an equation with two variables, the display shows the points (ordered pairs) that satisfy the equation. The display should look like a curve, line, or part of a line, depending on the situation.

Example:

Use a rectangular coordinate system to display the solution to $3x + y = 14$.

Solution:

First solve the equation for y.

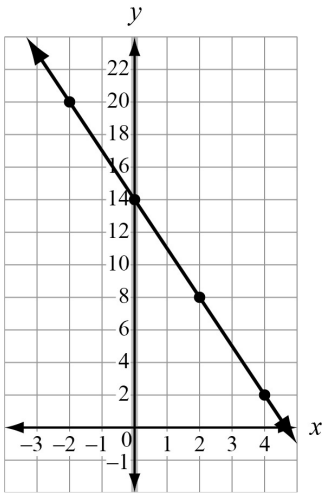
$$\begin{aligned} 3x + y &= 14 \\ 3x - 3x + y &= 14 - 3x \\ y &= 14 - 3x \end{aligned}$$

After transforming the equation, the result is $y = 14 - 3x$. We will need to determine some ordered pairs of numbers for x and y that satisfy the equation. A good way to do this is to organize your findings in an input/output table with a column for x and a column for y, as seen below.

x	$14 - 3x$	y
–1	14 – 3(–1)	17
0	14 – 3(0)	14
1	14 – 3(1)	11
2	14 – 3(2)	8

After choosing some numbers for x, replace x with those numbers and solve for y. Use the numbers in the first and last columns of the table as the coordinates of the points on your graph. Connect the points, unless the numbers you chose are the only ones you may use.

Here is the graph of $3x + y = 14$.



3. For an inequality, the display shows the points (ordered pairs) that satisfy the inequality. The display is usually a shaded half-plane that may or may not include values on its boundary line. If values on the boundary line are not included (<, >), then a dashed line is drawn. If the boundary line is included (≥, ≤), the line is solid. Depending on the situation, parts of the half-plane may not be included in the solution and therefore are not shaded.

Example:

Use a rectangular coordinate system to display the solution to $3x + y > -1$.

Solution:

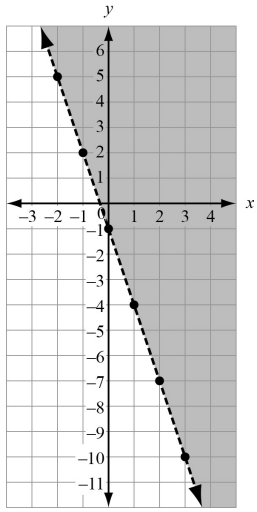
First, solve the inequality for y.

$$3x + y > -1$$
$$3x - 3x + y > -1 - 3x$$
$$y > -1 - 3x \quad \text{or} \quad y > -3x - 1$$

After transforming the equation the result is $y > -3x - 1$. We will need to determine some ordered pairs of numbers for x and y that fit the boundary line for this inequality. The equation $y = -3x - 1$ would be that boundary line. A good way to do this is to organize your findings in an input/output table with a column for x and a column for y, as seen below.

x	$-3x - 1$	y
–1	–3(–1) – 1	2
0	–3(0) – 1	–1
1	–3(1) – 1	–4
2	–3(2) – 1	–7

Use the numbers in the first and last columns of the table as the coordinates of the points on your boundary line. Use a broken line to connect the dots, unless the numbers you chose are the only ones you may use. The line is a broken line because the inequality is strictly greater than. The graph of $3x + y > -1$ is shown. There is shading above the line because the inequality is for points "greater than" the boundary line.



Example:

Use a rectangular coordinate system to display the solution to $y + 2 \leq x$.

Solution:

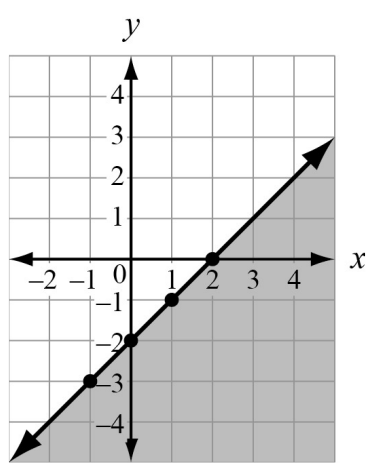
First, solve the inequality for y.

$$\begin{aligned} y + 2 &\leq x \\ y + 2 - 2 &\leq x - 2 \\ y &\leq x - 2 \end{aligned}$$

After transforming the equation, the result is $y \leq x - 2$. We will need to determine some ordered pairs that fit the boundary line for this inequality. The equation $y = x - 2$ would be that boundary line. A good way to do this is to organize your findings in an input/output table with a column for x and a column for y, as seen below.

x	$x - 2$	y
–1	(–1) – 2	–3
0	(0) – 2	–2
1	(1) – 2	–1
2	(2) – 2	0

Use the numbers in the first and last columns of the table as the coordinates of the points on your boundary line. Use a solid line to connect the points, unless the numbers you chose are the only ones you may use. The line is a solid line because the inequality is less than or "equal to" the boundary line. The graph of $y \leq x - 2$ is shown below. There is shading below the boundary line because the inequality is for points "less than" or on the boundary line.



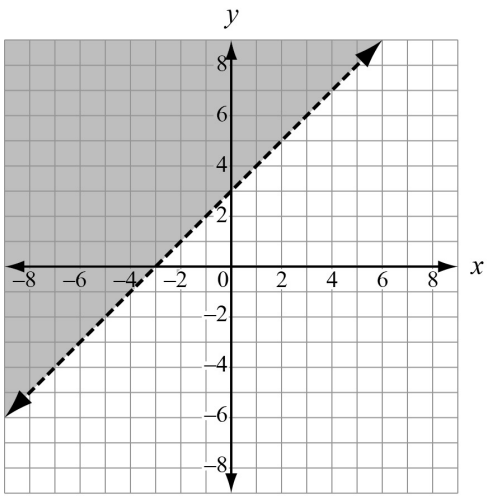
4. A pair of two-variable inequalities can be graphed by finding the region where the shaded half-planes of both inequalities overlap.

Example:

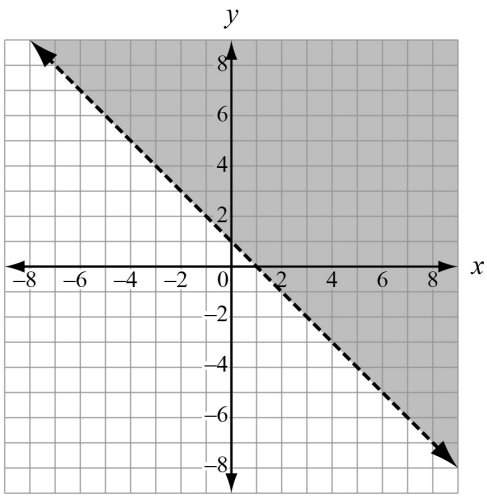
Graph the solution of $y > x + 3$ and $y > -x + 1$.

Solution:

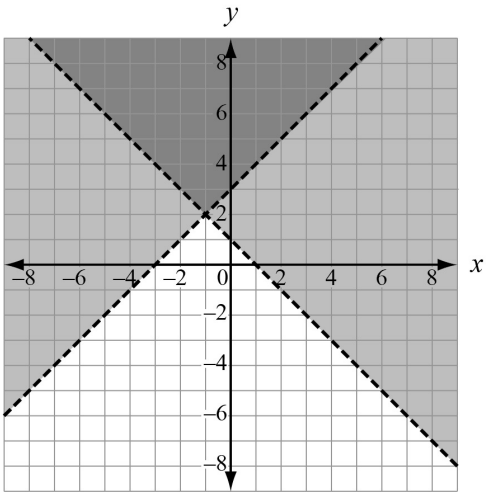
The left graph shows $y > x + 3$, the right graph $y > -x + 1$, and the graph below them shows the solution where the diagonal stripes are. Both boundary lines are dashed because the inequalities were strictly "greater than." All the points (ordered pairs of coordinates) in the region of the diagonal stripes satisfy both inequalities.



$y > x + 3$



$y > -x + 1$



Important Tips

- Know when to use a number line (one variable) and when a rectangular coordinate graph (two variables) is used.
- Notice when the relationship is "strictly" an inequality—the starting point is hollow or the boundary line is dashed.

REVIEW EXAMPLE

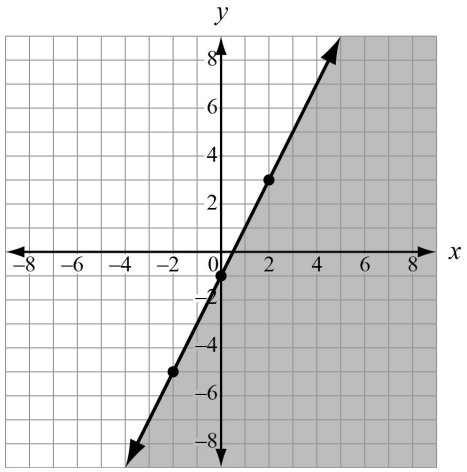
1) Graph the solution region for $y \leq 2x - 1$.

Solution:

First graph the boundary line, which is solid because the inequality is "less than or equal." The boundary line would be the equation $y = 2x - 1$. We can make a table of three or more rows and the numbers in the rows can become the coordinates of points on the boundary line.

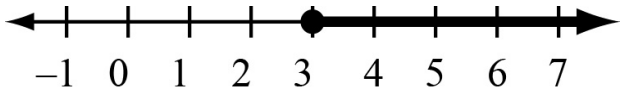
x	***y***	***(x, y)***
–2	2(–2) – 1 = –5	(–2, –5)
0	2(0) – 1 = –1	(0, –1)
2	2(2) – 1 = 3	(2, 3)

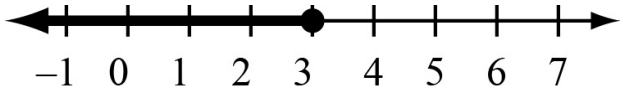
Next, decide which side of the boundary line to shade. Typically you can test a point above the line. If it satisfies the inequality, then shade *above* the line. If the test point does not work, shade *below* the line. We could test (0, 0): Is $0 \leq 2(0) - 1$? No, so the shading does not belong above the line. The graph for $y \leq 2x - 1$ is represented below.

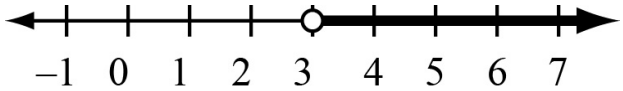


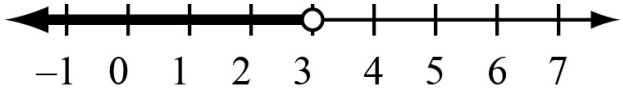
EOCT Practice Items

1) Which graph represents the solution to $x > 3$?

A. 

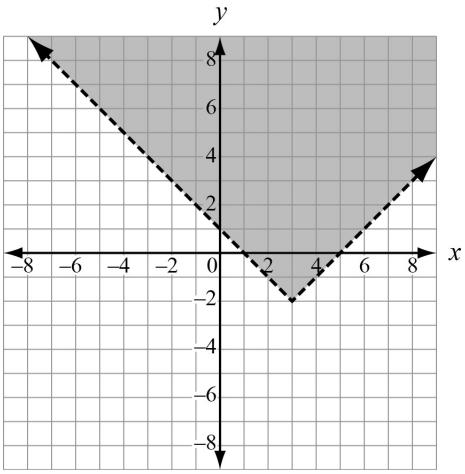
B. 

C. 

D. 

[Key: C]

2) Which pair of inequalities is shown in the graph?



A. $y > -x + 1$ and $y > x - 5$

B. $y > x + 1$ and $y > x - 5$

C. $y > -x + 1$ and $y > -x - 5$

D. $y > x + 1$ and $y > -x - 5$

[Key: A]