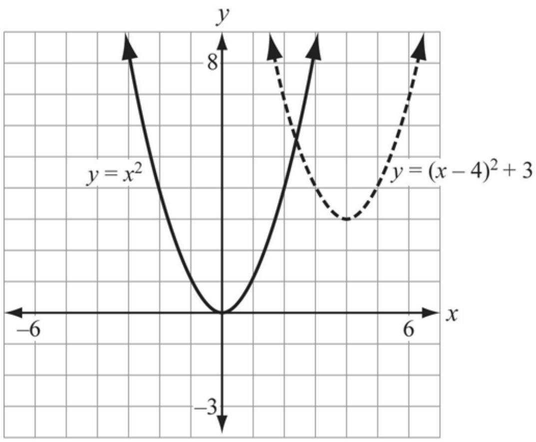
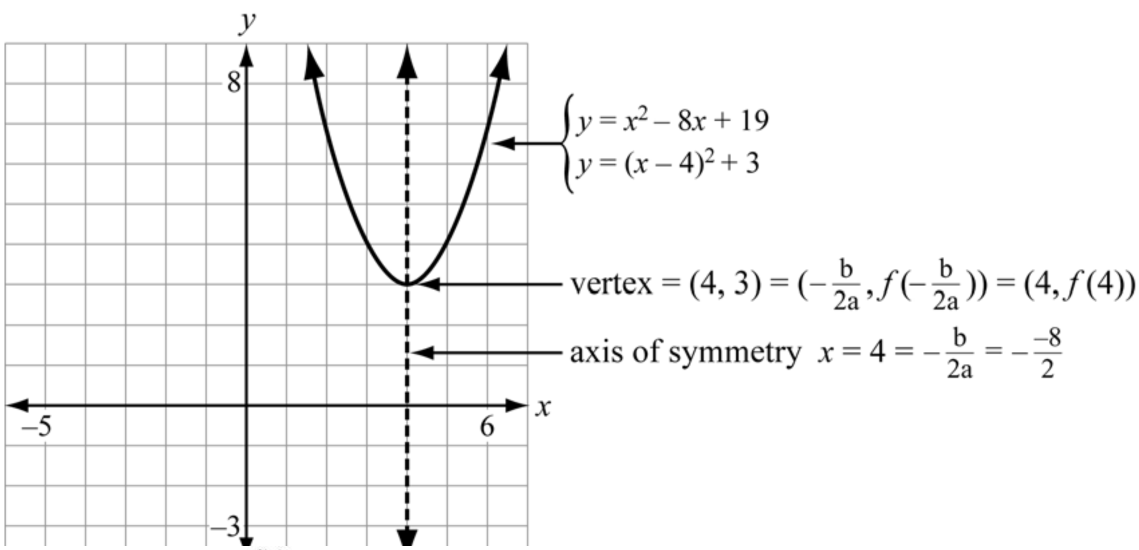
Math II EOCT Cram Sheet

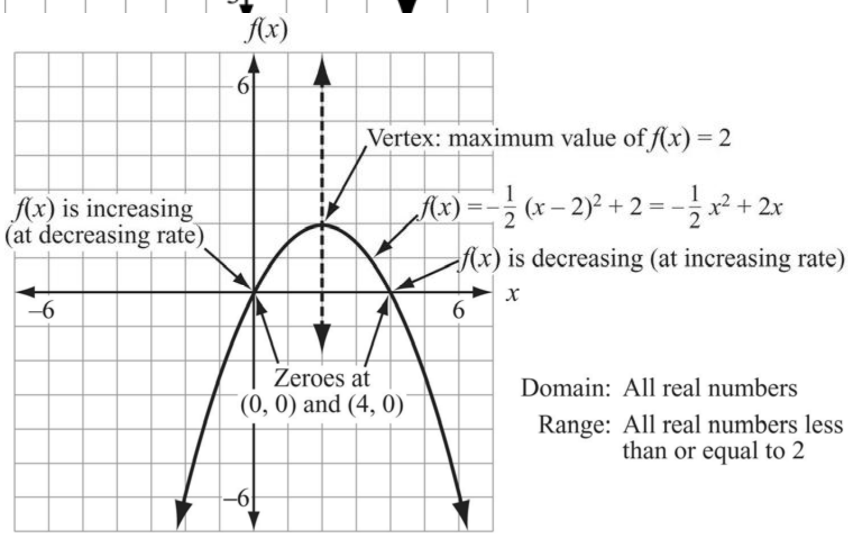
Quadratics:

Horizontal & Vertical Shift



Vertex & Axis of Symmetry



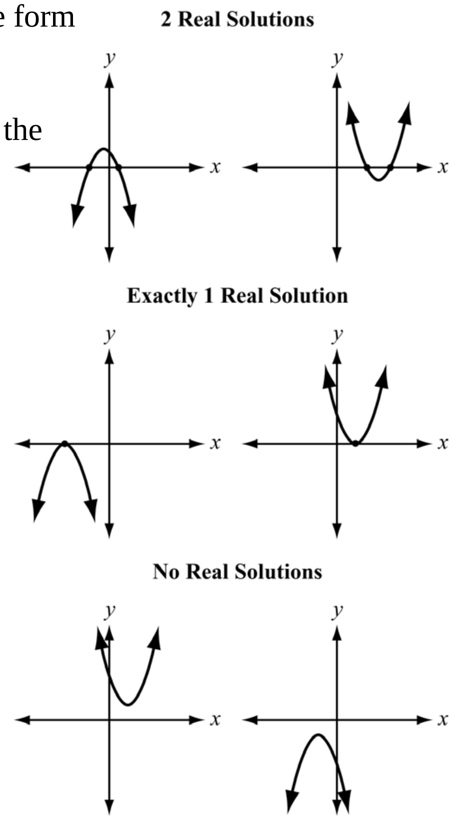


The ***quadratic formula*** gives a general solution to ***any*** quadratic equation of the form

$0 = ax^2 + bx + c$.

The value of the ***discriminant***, $b^2 - 4ac$, determines the number and character of the solutions to a quadratic equation.

- If $b^2 - 4ac > 0$, the equation has ***two*** real solutions.
- If $b^2 - 4ac = 0$, the equation has ***exactly one*** real solution.
- If $b^2 - 4ac < 0$, the equation has ***no*** real solutions; there are two ***complex*** solutions.



Imaginary & Complex Numbers:

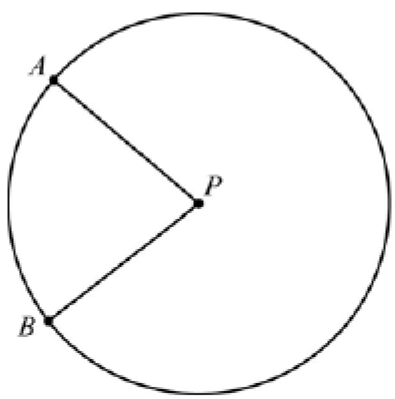
The basis of the complex number system is the solution to the equation $x^2 = -1$. There is no real number that, when multiplied by itself, is equal to –1. This problem was solved by defining the imaginary number i such that $i^2 = -1$; i.e., $i = -1$.

Partial Sum Formula – Arithmetic Series:

$$\frac{n(t_1 + t_n)}{2}$$

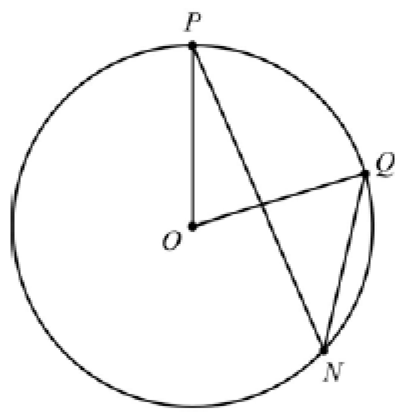
Circle Vocabulary & Theorems:

The measure of a minor arc of a circle is equal to the measure of the corresponding central angle.



$$m\angle APB = m\overset{\frown}{AB}$$

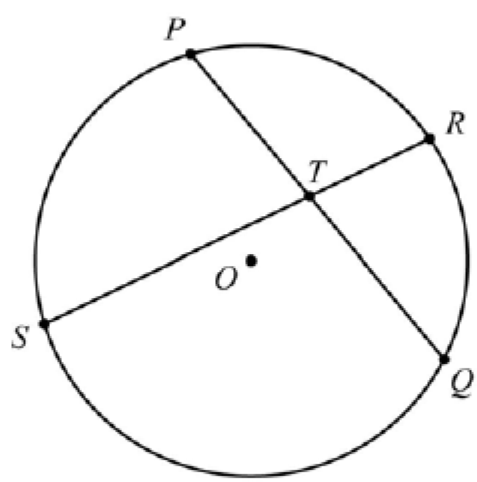
The measure of an angle inscribed in a circle, that is, an angle that has its vertex on the circle, is half the measure of the corresponding minor arc, as shown below.



$$m\angle PNQ = \frac{1}{2} m\angle \overset{\frown}{PQ}$$

$$m\angle POQ = m\overset{\frown}{PQ} = 2(m\angle PNQ)$$

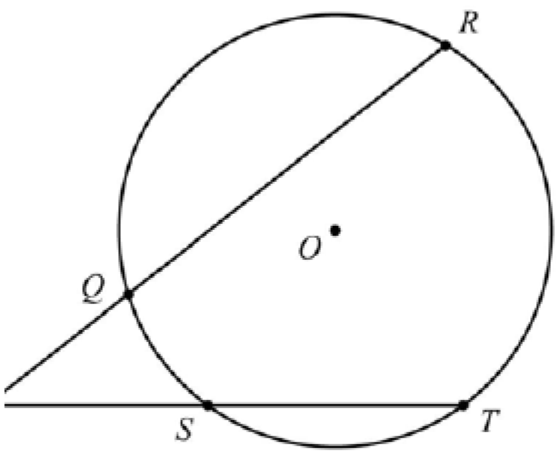
If two chords intersect, the angles created have a measure that is equal to half of the sum of the corresponding arc measures, as shown in this circle.



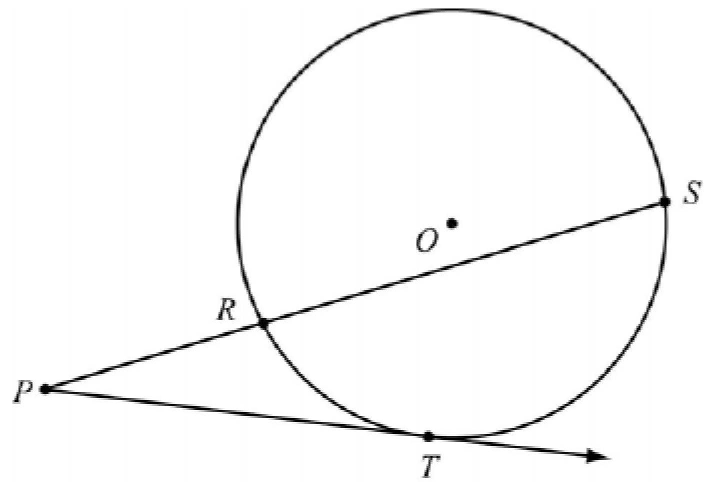
$$m\angle PTR = m\angle STQ = \frac{1}{2}\left(m\overset{\frown}{PR} + m\overset{\frown}{SQ}\right)$$

$$m\angle PTS = m\angle RTQ = \frac{1}{2}\left(m\overset{\frown}{PS} + m\overset{\frown}{RQ}\right)$$

The angle created by two secants or by a secant and a tangent has a measure that is equal to half the difference of the corresponding arc measures, as shown in these circles.

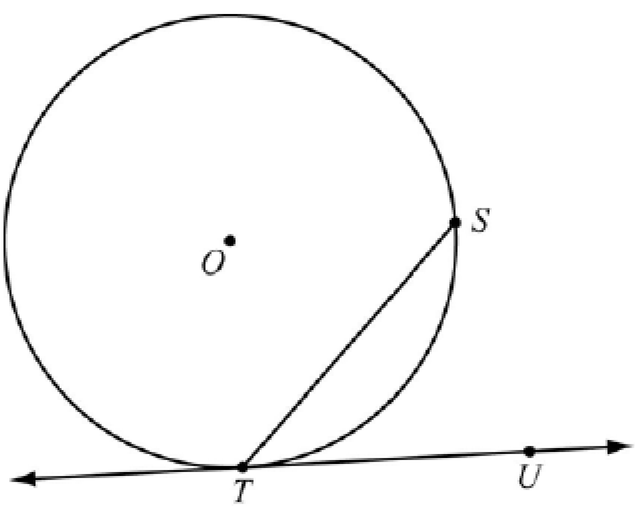


$$m\angle RPT = \frac{1}{2}\left(m\overset{\frown}{RT} - m\overset{\frown}{QS}\right)$$



$$m\angle SPT = \frac{1}{2}\left(m\overset{\frown}{ST} - m\overset{\frown}{RT}\right)$$

The angle created by a tangent and a chord has a measure that is equal to half the measure of its intercepted arc, as shown in this circle.



$$m\angle STU = \frac{1}{2}m\overset{\frown}{ST}$$

The length of an arc that corresponds to a central angle with a measure of θ degrees in a circle with a radius r is $\frac{\theta}{360}$ times the circumference of the circle; i.e., $\frac{\theta}{360} \cdot 2\pi r = \frac{\theta \pi r}{180}$.

The area of a sector that corresponds to a central angle with a measure of θ degrees in a circle with a radius r is $\frac{\theta}{360}$ times the area of the circle; i.e., $\frac{\theta}{360} \cdot \pi r^2$.

Review Mean Absolute Deviation & Standard Deviation Formulas

Piecewise Functions:

Remember that the ***domain*** of a function is the set of input numbers, and the ***range*** is the set of output numbers. In a piecewise function, the input number determines which equation to use to find the output number.

A ***point of discontinuity*** is a point where there is a break or a gap in the graph. A graph is said to be discontinuous when there is a break or a gap in it.

Absolute Value Equations:

$$|3x-4|=11$$
$$3x-4=11 \quad \text{or} \quad 3x-4=-11$$
$$3x=15 \quad \text{or} \quad 3x=-7$$
$$x=5 \quad \text{or} \quad x=-\frac{7}{3}$$

Absolute Value Inequalities:

$$|2x+5|<13$$
$$-13<2x+5<13$$
$$-18<2x<8$$
$$-9<x<4$$

$$|2x+5|>13$$
$$2x+5<-13 \quad \text{or} \quad 2x+5>13$$
$$2x<-18 \quad \text{or} \quad 2x>8$$
$$x<-9 \quad \text{or} \quad x>4$$

Exponential Functions:

There are five basic ***properties of exp***

a. $a^n a^m = a^{n+m}$

Example:

$2^3 \bullet 2^5 = 2^{3+5} = 2^8 = 256$

b. $(a^n)^m = a^{n \cdot m}$

Example:

$(3^2)^3 = 3^{2 \cdot 3} = 3^6 = 729$

c. $a^0 = 1$

Example:

$5^0 = 1$

d. $\frac{a^n}{a^m} = a^{n-m}$

Example:

$\frac{2^5}{2^3} = 2^{5-3} = 2^2 = 4$

e. $a^{-n} = \frac{1}{a^n}$

Example:

$$25^{x-2} = 125^{4x}$$
$$(5^2)^{x-2} = (5^3)^{4x}$$
$$5^{2(x-2)} = 5^{3(4x)}$$
$$5^{2x-4} = 5^{12x}$$

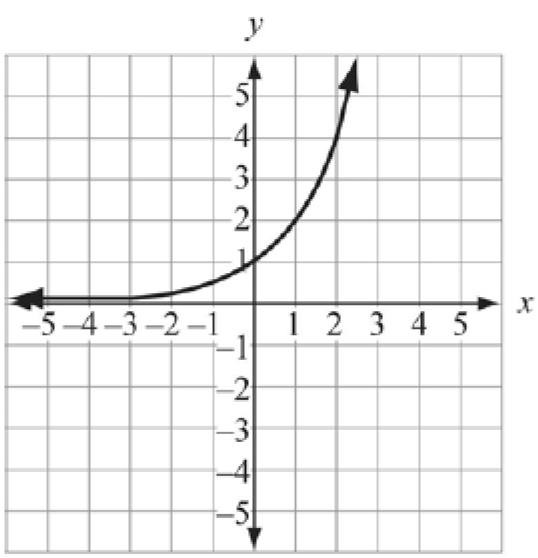
Now that the bases are the same, the exponents are equal.

$$2x - 4 = 12x$$
$$10x = -4$$
$$x = -\frac{2}{5}$$

An ***exponential growth function*** can be written in the form $f(x) = ab^x$, where $a > 0$ and $b > 1$. An ***exponential decay function*** can be written in the form $f(x) = ab^x$, where $a > 0$ and $0 < b < 1$.

Exponential Growth:

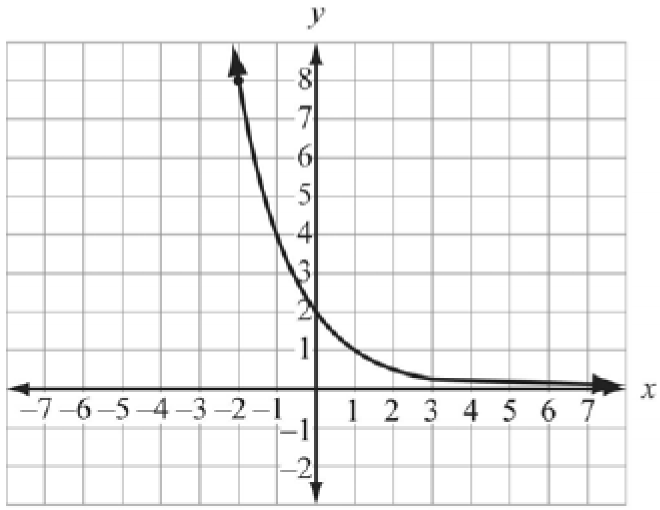
This is the graph of the function.



Notice the ***end behavior*** of the graph. As the value of x increases, the graph moves up to the right, and the value of y increases without bound. As the value of x decreases, the graph moves down to the left and approaches the x-axis or $y = 0$, which is called an ***asymptote***.

Exponential Decay:

This is the graph of the function.



Notice the end behavior of the graph. As the value of x increases, the graph moves down to the right and approaches the x-axis or the asymptote $y = 0$. As the value of x decreases, the graph moves up to the left. As the value of x gets very small, the value of y increases without bound.

The domain for this function is all real numbers. The range is $y > 0$.

One, two, three, or all four of the transformations shown below can be applied to an exponential function.

a. The graph of $f(x) = b^x$ rewritten in the form $f(x) = ab^{x-h} + k$ is ***translated horizontally*** to the right h units if $h > 0$ or to the left h units if $h < 0$.

b. The graph of $f(x) = b^x$ rewritten in the form $f(x) = ab^{x-h} + k$ is ***translated vertically*** up k units if $k > 0$ or down k units if $k < 0$.

c. The graph of $f(x) = b^x$ rewritten in the form $f(x) = ab^{x-h} + k$ is ***vertically stretched*** if $a > 1$. The graph is ***vertically shrunk*** if $0 < a < 1$.

d. The graph of $f(x) = b^x$ is ***reflected across the x-axis*** if b^x is multiplied by -1. The graph of $f(x) = b^x$ is ***reflected across the y-axis*** if the exponent x in b^x is multiplied by -1.

A ***geometric sequence*** can be written as an exponential function with a domain that consists of positive integers. In a geometric sequence, the ratio of any term to its preceding term is constant. The ***constant ratio*** is called the ***common ratio*** and is the base of the associated exponential function. The *n*th term of a geometric sequence with the first term a_1 and with a common ratio *r* is $f(n) = ar^{n-1}$.

The first three terms of the sequence are found using the function, as shown below.

First term:

$f(n) = 2(3)^{n-1}$

$f(1) = 2(3)^{1-1} = 2(3)^0 = 2(1) = 2$

Second term:

$f(n) = 2(3)^{n-1}$

$f(2) = 2(3)^{2-1} = 2(3)^1 = 2(3) = 6$

Third term:

$f(n) = 2(3)^{n-1}$

$f(3) = 2(3)^{3-1} = 2(3)^2 = 2(9) = 18$

The ***natural base e*** is an irrational number. It is defined as $\lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$. As *n* increases, the value of the expression $\left(1 + \frac{1}{n}\right)^n$ approaches *e*, and $e \approx 2.718281828459$. The natural base *e* is used in the formula $A = Pe^{rt}$ to calculate continuously compounded interest.

Composition of Functions:

The ***composition of functions*** is a method of combining functions. The composition of functions $f(x)$ and $g(x)$ is written $(f \circ g)(x)$ and is defined by $(f \circ g)(x) = f[g(x)]$.

Example:

Let $f(x) = x^2$ and $g(x) = 2x + 3$. Find $(f \circ g)(x)$.

$$(f \circ g)(x) = f[g(x)]$$

Substitute $2x + 3$ for $g(x)$ $\quad = f(2x + 3)$

It was given that $\quad f(x) = x^2$

So it follows that $\quad f(2x+3) = (2x+3)^2$

That simplifies to $\quad = 4x^2 + 12x + 9$

So $\quad (f \circ g)(x) = 4x^2 + 12x + 9$

An ***inverse function*** reverses the process of the original function. A function maps the input values onto the output values. An inverse function maps the output values onto the original input values. Switching the x-values and the y-values in an input-output table produces an inverse function. The same thing is true if the x and the y variables are switched in an equation. The inverse of a function is denoted by f^{-1} and is read "f inverse."