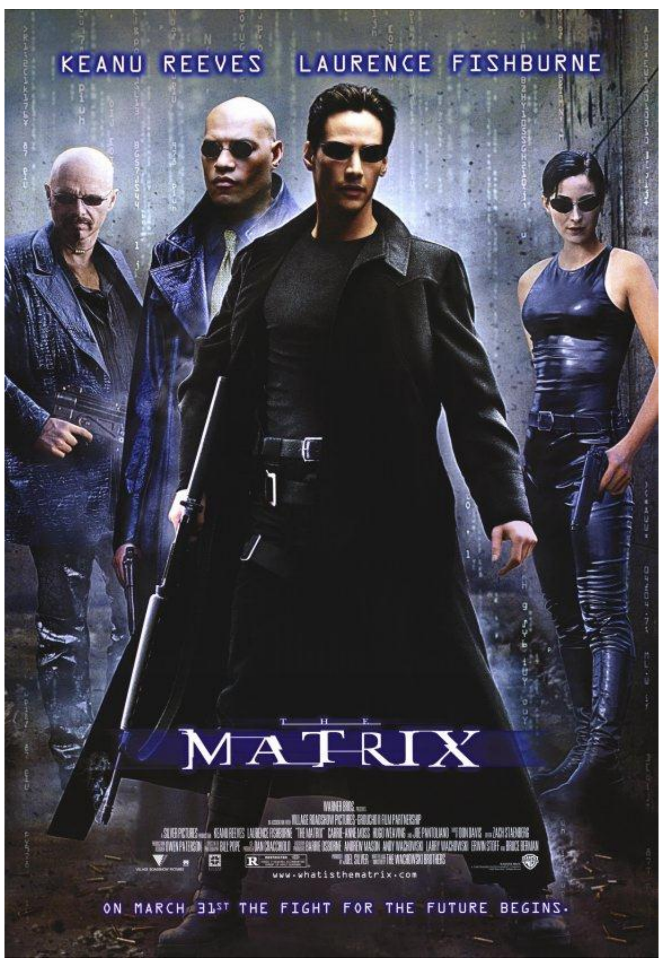
Name: ______________________________ Period: __________________

ALGEBRA II

UNIT IV: MATRIX OPERATIONS

Unit Notes Packet



Algebra II

Unit 4 Plan: This plan is subject to change at the teacher's discretion.

Section	**Topic**	**Formative Work**	**Due Date**
4.1 & 4.2	Matrix Properties	Pg 188 #9-21 Pg 196 #1-11, 24, 25, 31	10/24
4.3	Multiplying Matrices	4.3 Practice #'s 1-14, 15, 17, 19	10/25
4.5.1	Determinants	4.5 Skills Practice WS	10/26
4.5.2	Cramer's Rule	4.5 Practice WS #'s 1-15 odd, 16-26 all	10/27
Review	4.1 – 4.5	Start Review Packet	
Review	4.1 – 4.5	Finish Review Packet	11/1
Test	4.1 – 4.5	Eat Halloween Candy	

UNIT 4 TEST DAY – 11/1/2011

Section 4.1: Matrix Properties

Warm-up: Solve using the distributive property.

A. 5(12 – 4)

B. -3(7 – 9 + 4)

C. 6(2*x* – 9)

D. $-3(-6x^2 - 5x + 1)$

Notes: Matrix Properties and Matrix Addition/Subtraction

What you see below is the matrix A.

$$A = \begin{bmatrix} 6 & 2 & -1 \\ -2 & 0 & 5 \end{bmatrix}$$

Matrix Dimensions: _______ by __________

Dimensions of A: ___________

The plural of matrix is _____________.

Sometimes we want to add or subtract matrices.

Can you add matrices of different sizes? What do you think? _________________________

Practice: Add or subtract the matrices.

1. $\begin{bmatrix} 2 & 3 \\ -1 & 4 \\ 0 & -2 \end{bmatrix} + \begin{bmatrix} 5 & 1 \\ 12 & 2 \\ -3 & 16 \end{bmatrix} =$

2. $\begin{bmatrix} 1 & -5 & 3 \end{bmatrix} - \begin{bmatrix} 4 & -10 & 2 \end{bmatrix} =$

3. $\begin{bmatrix} 5 & 6 & 5 \\ -3 & 0 & -1 \end{bmatrix} + \begin{bmatrix} 4 & 6 \\ 1 & -1 \\ 7 & 9 \end{bmatrix} =$ ____________________

4. $\begin{bmatrix} 1 \\ 5 \\ -2 \end{bmatrix} + \begin{bmatrix} 7 \\ 2 \\ 0 \end{bmatrix} =$

5. $\begin{bmatrix} 1 & 2 \\ 6 & 5 \end{bmatrix} - \begin{bmatrix} 7 & 1 \\ 4 & 12 \end{bmatrix} =$

6. $\begin{bmatrix} 3 & -4 & 12 \\ 4 & 5 & -1 \\ 0 & 10 & 6 \end{bmatrix} - \begin{bmatrix} 1 & 4 \\ 0 & 7 \end{bmatrix} =$

Notes: Multiplying by a *Scalar*

A *scalar* is ____

Why do they call it a *scalar?* ______________________________________

7. $3\begin{bmatrix} 3 & 4 & 5 \\ -2 & 0 & -1 \end{bmatrix} =$

Multiplying by a scalar is very similar to using the ____________________________

8. $\frac{1}{2}\begin{bmatrix} 2 & -6 \\ 1 & 3 \end{bmatrix} =$

Notes: Solving Matrix Equations

Equivalent Matrices have the same ______________ and the same ___________________.

Write the equivalent matrix.

9. $\begin{bmatrix} 5 & 0 \\ -\frac{4}{4} & \frac{3}{4} \end{bmatrix} =$

We can use our knowledge about equivalent matrices to __________________________________

Practice: *Solve the matrix equations for each variable.*

10. $\begin{bmatrix} x & -1 \\ 3 & 4y \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 3 & 12 \end{bmatrix}$

x =

y =

11. $\begin{bmatrix} 4 & -3 \\ 8 & -7 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} -5 & x \\ -7 & 7 \\ 4 & -9 \end{bmatrix} = \begin{bmatrix} -1 & -8 \\ y & 0 \\ 5 & -7 \end{bmatrix}$

12. $\begin{bmatrix} 3x & 18 \\ -y & 5 \end{bmatrix} = \begin{bmatrix} -21 & 2z \\ 12 & 5 \end{bmatrix}$

13. $2\left(\begin{bmatrix} 3x & -1 \\ 8 & 5 \end{bmatrix} + \begin{bmatrix} 4 & 1 \\ -2 & -y \end{bmatrix}\right) = \begin{bmatrix} 26 & 0 \\ 12 & 8 \end{bmatrix}$

Section 4.3: Matrix Multiplication

Warm-up: *Solve the matrix equations for each variable.*

A.

$$\begin{bmatrix} 4x \\ -5y \\ 27 \end{bmatrix} = \begin{bmatrix} 76 \\ -85 \\ -9z \end{bmatrix}$$

B.

$$\begin{bmatrix} 8 & 7 \\ -5 & -1 \\ 3 & 3 \end{bmatrix} + \begin{bmatrix} -5 & x \\ -7 & 7 \\ 4 & -9 \end{bmatrix} = \begin{bmatrix} 3 & 15 \\ -3y & 6 \\ 7 & -6 \end{bmatrix}$$

Notes: Multiplying Matrices

Can you multiply any two matrices? ____

So when can you multiply two matrices? ____

Let's say A and B are two matrices that we want to multiply. We are interested in the matrix dimensions of A and the matrix dimensions of B.

$$\underset{m \times n}{A} \cdot \underset{n \times p}{B} = \underset{m \times p}{AB}$$

Below each matrix, we write the matrix dimensions.

If you want to multiply matrices, ___from matrix A and ___ from matrix B must be same.

- If the number of columns in matrix A (n) is equal to the number of rows in matrix B (n), you can multiply the matrices.
- If they do not match up, ____

If you can multiply the matrices, you will find the matrix AB. So, $A \cdot B =$ ______
The dimension of AB should be m x p.

So *AB* should have the number of rows of A and the number of columns of B.

Practice: *First, state if matrix A and matrix B can be multiplied. If they can, write in the dimension of AB.*

1.
Can you multiply? (Circle 1: Yes/No)

$$\underset{3 \times 2}{A} \cdot \underset{2 \times 4}{B} = \underset{\_\_ \times \_\_}{AB}$$

2.
Can you multiply? (Circle 1: Yes/No)

$$\underset{4 \times 6}{A} \cdot \underset{6 \times 1}{B} = \underset{\_\_ \times \_\_}{AB}$$

3.
Can you multiply? (Circle 1: Yes/No)

$$\underset{1 \times 4}{A} \cdot \underset{1 \times 4}{B} = \underset{\_\_ \times \_\_}{AB}$$

4.
Can you multiply? (Circle 1: Yes/No)

$$\underset{2 \times 2}{A} \cdot \underset{2 \times 2}{B} = \underset{\_\_ \times \_\_}{AB}$$

Practice: Multiply the Matrices.

5.

$$\begin{bmatrix} -2 & 3 \\ 1 & -4 \\ 6 & 0 \end{bmatrix} \bullet \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} (-2)(-1)+(3)(-2) & (-2)(3)+(3)(4) \\ (1)(-1)+(-4)(-2) & (1)(3)+(-4)(4) \\ (6)(-1)+(0)(-2) & (6)(3)+(0)(4) \end{bmatrix} = \begin{bmatrix} \quad & \quad \\ \quad & \quad \\ \quad & \quad \end{bmatrix}$$

Notes: How to multiply Matrices

<u>*Step 1*</u>: Initially, before you do anything else, the first thing you *must* do, prior making any other moves, ____

<u>*Step 2*</u>: Now, you take the __________ of the first matrix and multiply each entry by the corresponding place of the ________________ of the next matrix and add them together…

FIND THE RHYTHM!

Practice: Multiply the following matrices. If it is not possible, write "not possible".

6.

$$\begin{bmatrix} 6 & -2 \\ 1 & 4 \\ 0 & 5 \end{bmatrix} \bullet \begin{bmatrix} -4 & -2 & 5 \\ 4 & -6 & -1 \end{bmatrix} =$$

7.

$$\begin{bmatrix} -\frac{1}{6} & \frac{1}{2} & -\frac{1}{3} \end{bmatrix} \bullet \begin{bmatrix} 12 \\ 0 \\ -12 \end{bmatrix} =$$

8.

$$\begin{bmatrix} 1 & -4 \\ 3 & -2 \end{bmatrix} \bullet \begin{bmatrix} 4 & -1 \\ 0 & -3 \end{bmatrix} =$$

9.

$$\begin{bmatrix} -4 & 8 & 2 \end{bmatrix} \bullet \begin{bmatrix} -5 & 1 \\ 8 & -3 \end{bmatrix} =$$

10.

$$\begin{bmatrix} 7 & 3 & -2 \\ -1 & 4 & 9 \\ 4 & -5 & 9 \end{bmatrix} \bullet \begin{bmatrix} 1 & 2 & -8 \\ 5 & -3 & 1 \\ 4 & 8 & -7 \end{bmatrix} =$$

11.

$$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \bullet \begin{bmatrix} -2 & -1 \\ 0 & 4 \\ -4 & 0 \end{bmatrix} =$$

Section 4.5.1: Determinants

Warm-up: *Multiply the following matrices.*

A.

$$\begin{bmatrix} 0 & 3 \\ 2 & -5 \\ -1 & 2 \end{bmatrix} \bullet \begin{bmatrix} -7 & 6 & 5 \\ 4 & -9 & -5 \end{bmatrix} =$$

B.

$$\begin{bmatrix} 7 & -3 & 2 & -9 \end{bmatrix} \bullet \begin{bmatrix} 3 \\ -2 \\ 4 \\ 6 \end{bmatrix} =$$

By the end of today you will know how to find the area of a triangle using determinants.

Notes: The Determinant of a 2 x 2 Matrix

- The determinant of a matrix is found by multiplying the diagonals down and subtracting the product of the diagonals up. "*Multiply diagonals down minus multiply diagonals up*."

Wait, what? Take a look at the problem below.

$$\det\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} =$$

$$\det\begin{pmatrix} 1 & 3 \\ 2 & 5 \end{pmatrix} = \begin{vmatrix} 1 & 3 \\ 2 & 5 \end{vmatrix} = \_\_\_\_\_\_\_\_\_\_ - \_\_\_\_\_\_\_\_\_\_ = \_\_\_\_ - \_\_\_\_ =$$

Practice: Find the determinant of each 2 x 2 matrix.

1.

$$\det\begin{pmatrix} 3 & -4 \\ 2 & -6 \end{pmatrix} =$$

2.

$$\begin{vmatrix} -8 & 3 \\ -11 & -7 \end{vmatrix} =$$

3.

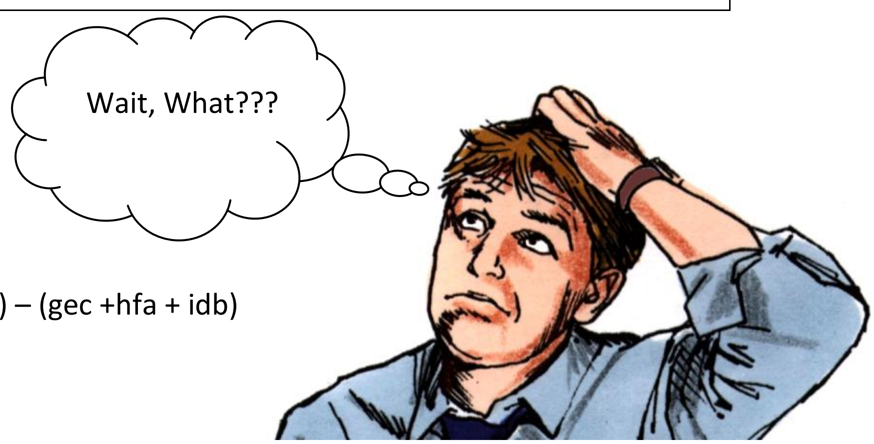
$$\begin{vmatrix} 4 & 5 \\ -7 & 9 \end{vmatrix} =$$

Notes: Finding the determinant on a 3 x 3 matrix

Step 1: Rewrite the first 2 columns to the right of the matrix.

Step 2: Follow the pattern below.

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$



$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} \begin{matrix} a & b \\ d & e \\ g & h \end{matrix}$$ = (aei + bfg +cdh) – (gec +hfa + idb)

Here's the secret: ____

Practice: Find the determinant of each 3 x 3 matrix.

4.

$$\begin{vmatrix} 2 & -1 & 3 \\ -2 & 0 & 1 \\ 1 & 2 & 4 \end{vmatrix}$$

5.

$$\begin{vmatrix} -5 & 4 & 3 \\ 2 & 1 & 0 \\ 5 & -1 & -2 \end{vmatrix}$$

6.

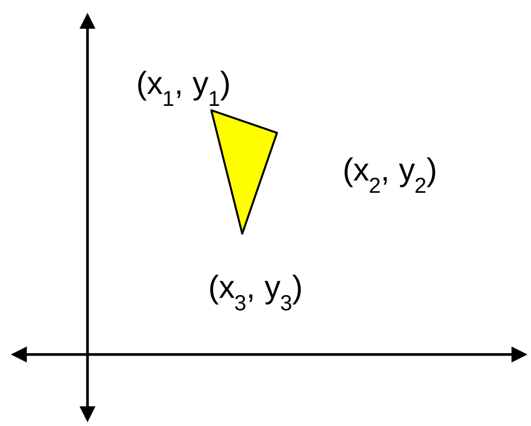
$$\begin{vmatrix} -7 & 1 & 4 \\ -3 & 1 & 3 \\ 9 & -2 & -1 \end{vmatrix}$$

Notes: Area of a triangle, using determinants.

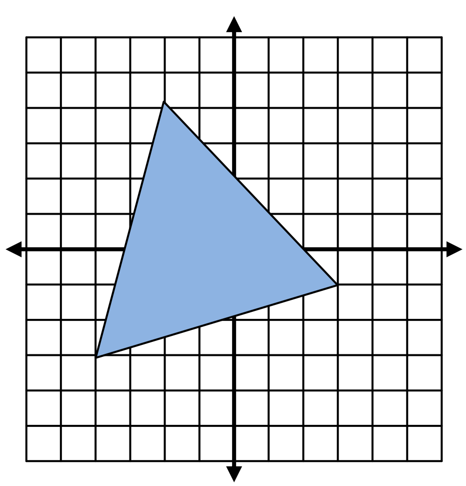
The area of a triangle, with vertices at the points (x_1, y_1), (x_2, y_2), (x_3, y_3) is given by…

Area = $\pm\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

The symbol $\pm$ is used to make the final answer_____________, because area is always ____________________.



Practice: Find the area of the triangle below by finding a determinant.



Section 4.5.2: Cramer's Rule

Warm-up: Find the determinants of each of the following matrices.

A.

$$\det\begin{pmatrix} 5 & -3 \\ 7 & -5 \end{pmatrix} =$$

B.

$$\begin{vmatrix} 10 & 2 \\ -9 & 8 \end{vmatrix} =$$

C.

$$\begin{vmatrix} -2 & 5 & 0 \\ 2 & 2 & 4 \\ 3 & -9 & -6 \end{vmatrix}$$

No, not that Kramer…

Notes: Cramer's Rule with systems of 2 variables.

Remember in Unit 3 when we were solving systems of equations? Well there is actually an easy way to solve systems of equations using the determinants we just learned about.

Just use Cramer's Rule!

Cramer's rule..
Given a generic system of equations such as..

$$\begin{cases} a_1x + b_1y = c_1 \\ a_2x + b_2y = c_2 \end{cases}$$

We want to find *x* and *y* using Cramer's Rule. So we write three determinants…

D = $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$ D_x = $\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}$ D_y = $\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}$

D is the determinant with the coefficients of *x* and *y*.

D_x – replace the *x*- coefficients with the constant column.

D_y – replace the *y*- coefficients with the constant column.

Once you find each of the three determinants, getting x and y is simple.

x =

y =

Practice: Find *x* and *y* using Cramer's Rule.

1.
2x + y = 3
5x + 6y = 4

2.
-2x – 6y = 6
3x + 9y = = -9

Yes, this is the Cramer we are talking about…

Notes: Using Cramer's Rule for systems of 3 equations.

$$\begin{cases} a_1x + b_1y + c_1z = d_1 \\ a_2x + b_2y + c_2z = d_2 \\ a_3x + b_3y + c_3z = d_3 \end{cases}$$

$$x = \frac{D_x}{D} \qquad y = \frac{D_y}{D} \qquad z = \frac{D_z}{D}$$

D is still the determinant with the coefficients of x and y and z.

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$D_x = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix} \qquad D_y = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix} \qquad D_z = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$$

D_x = Replace the x-coefficients with the constant column.

D_y = Replace the y-coefficients with the constant column.

D_z = Replace the z-coefficients with the constant column.

Practice: Solve using Cramer's Rule.

3.

$$x + 2y - 3z = -2$$
$$x - y + z = -1$$
$$3x + 4y - 4z = 4$$

4.

$x + 2y + z = 9$

$x + y + z = 3$

$5x - 2z = -1$